Bézier curves are parametric curves resulting from parametric equations.

Parametric equations express a set of quantities as explicit functions of a number of "parameters".

A Bézier curve takes as input the coordinates in x and y, and z (3D case) plus the variable t (time it takes to traverse the curve), and outputs a new point (red dot).

Case 1: A linear Bézier takes as input two points (A (magenta dot) and B (blue dot)) and outputs a new point W (red dot) (linear interpolation).

The equation to get a third point C in the middle of A and B:

W = 0.5A + 0.5B

The parametric expression of C results in the graphing of the parametric curve at points (x, y) regarding various t’s:

W = (1 - t)A + tB

Case 2: A Quadratic or 3rd degree Bézier results from extrapolating case 1, where a new control point C (cyan) is added, and the interpolation happens between A and B, and B and C. A further step of interpolation between the new points D and E (orange) result in the t parameter W (red dot).

D = ( 1 - t )A + tB

E = ( 1 - t )B + tC

W = ( 1 - t )D + tE

This is a system of equations, so we can substitute D and E directly into W without having to precompute them:

W = ( 1 - t )[ ( 1 - t )A + tB ] + t[ ( 1 - t )B + tC

W = ( 1 - t )^2A + 2( 1 - t )B + t^2C

Case 3: Parametric Cubic or degree 4 Bézier curve, which extrapolates from the first and second cases, and it could be taken to higher degrees too.

E = ( 1 - t )A + tB

F = ( 1 - t )B + tC

G = ( 1 - t )C + tD

H = ( 1 - t )E + tF

I = ( 1 - t )F + tG

W = ( 1 - t )H + tI

That is:

W = ( 1 - t )[ ( 1 - t )( 1 - t )A + tB + ( 1 - t )B + tC ] + t[ ( 1 - t )( 1 - t )B + tC + ( 1 - t )C + tD ]

W = ( 1 - t )^3A + 3( 1 - t )^2tB + 3( 1 - t )t^2C + t^3D

Rational Bézier Curves

We could add another parameter called "weight" to the equations, which corresponds to the strength each t point is discretely pulled towards the control point. The equation reflects the transformed t parameter regarding both the calculations on the effect of weighted control points (control point \* weight \* t), and the parametrized (between 0 and 1) calculations amongst the weights and the parameter t (w \* t), for each t being evaluated.

W = ( 1 - t )Aw0 + tBw1 / ( 1 - t )w0 + tw1